El Monte Union High School District

Course Outline

High School District

Title: Integrated Math 3 P	This course meets graduation requirements:	Department/Cluster Approva	1 Date
Transitional*(Eng. Dept. Only)			
Sheltered (SDAIE)*Bilingual*	 () English () Fine Arts () Foreign Language 		
AP**Honors**	() Health & Safety(X) Math		
Department: Math	() Physical Education		
Grade Level (s): <u>9-12</u>	 () Social Science () Elective 		
Semester Year X			
Based on Mathematics CCSS			

*Instructional materials appropriate for English Language Learners are required.

**For AP/Honors course attach a page describing how this course is above and beyond a regular course. Also, explain why this course is the equivalent of a college level class.

1. **Prerequisite(s):** C or better in Integrated Math 2 or equivalent.

2. Short description of course which may also be used in the registration manual:

In Integrated Math 3, students integrate and apply the mathematics they have learned from their earlier courses. This course includes standards from the conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Instructional time should focus on four critical areas: (1) apply methods from probability and statistics to draw inferences and conclusions from data; (2) expand understanding of functions to include polynomial, rational, and radical functions; (3) expand right triangle trigonometry to include general triangles; and (4) consolidate functions and geometry to create models and solve contextual problems.

3. Describe how this course integrates the schools ESLRs (Expected School-wide Learning Results): This section may be replaced with specific site ESLRS

All schools have ESLRS that refer to students as academic achievers, critical thinkers, and effective communicators. This course addresses the mentioned ESLRS.

4. Describe the additional efforts/teaching techniques/methodology to be used to meet the needs of English Language Learners:

The special needs of English language learners are met throughout the course in a number of ways:

- By using the Sheltered Instruction Observation Protocol (SIOP) or other researched based strategies that engage students in learning and communicating their thoughts in the four language domains.
- By probing prior knowledge to connect existing knowledge with knowledge to be learned.
- By teaching concepts for which English learners may not have a cultural reference, including obscure terms, and academic vocabulary.
- By defining abstract concepts in concrete terms, and using specific examples.
- By using graphic organizers and rubrics to set expectations and facilitate organization of thought.
- By using a variety of other visual aids during instruction, such as pictures, films, and realia.
- By encouraging students to express themselves in a variety of modalities.

5. Describe the interdepartmental articulation process for this course:

The study of mathematics in each year of high school leads directly to preparedness for college and career readiness. The skills learned in math are applied to other courses of study including science, social science, and Career Technical Education (CTE). Problem solving, communicating reasoning, modeling and data analysis that are used in mathematics prepare students to apply those same skills in all courses and in real-world scenarios.

6. Describe how this course will integrate academic and vocational concepts, possibly through connecting activities. Describe how this course will address work-based learning/school to career concepts:

Various projects and Performance Tasks that relate real-life problems are implemented throughout the course. These Performance Tasks are usually assigned at the end of each chapter to assess the ability of students to apply concepts learned throughout the chapter towards a real-world situation.

7. Materials of Instruction (Note: Materials of instruction for English Language Learners are required and should be listed below.)

A. Textbook(s) and Core Reading(s):

- Mathematics III: Common Core (Pearson) Teacher's Edition and Student Workbook.
- CPM Core Connections Integrated III (Course 6) Teacher's Edition and Student Workbook.

- B. Supplemental Materials and Resources:
 - Teacher made resources
 - Overhead transparencies or documents for projection
 - Extra practice worksheets
 - CPM materials
 - Teacher made resources and manipulatives
 - Materials found on-line: projects; performance tasks, problems of the week...

C. Tools, Equipment, Technology, Manipulatives, Audio-Visual:

- Protractors
- Compasses
- Graphing Calculators
- Projectors
- Document Readers

8. (See below and attached)

- **Objectives of Course:** The objective of this course is to extend the mathematics students learned in Integrated Math 1 and 2 with emphasis on the following:
 - o Probability and statistics to draw inferences and conclusions from data
 - Expand understanding of functions to include polynomial, rational, and radical functions
 - Expand right triangle trigonometry to include general triangles.
 - Consolidate functions and geometry to create models and solve contextual problems.

The Mathematical Practice Standards will be infused throughout the course together with the CCSS for mathematics. This attention to the Practice Standards as well as the Content Standards will ensure that students experience mathematics as coherent, useful, and logical and make use of the student's ability to make sense of problem situations. After completing this course successfully, students will be prepared to proceed to Pre-calculus or Statistics.

Unit detail including projects and activities including duration of units (see attached)

SEE ATTACHED DOCUMENT

- Indicate references to state framework(s)/standards (If state standard is not applicable then national standard should be used)
- Student performance standards

Guidelines for grading are:

- A 90-100%
- B 80-89%
- C 70-79%
- D 60-69%
- F 59% and below

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Evaluation/assessment/rubrics

- District assessments (Common Formative Benchmark Assessments)
- Site assessments to include formative assessments
- o Performance Tasks

Include minimal attainment for student to pass course

Students must attain <u>at least</u> 60% overall average for all assignments (Tests, Quizzes, Homework, Classwork, Notes, etc.) for the course.

Integrated Math 3 - Pearson (Algebra 2) Course Pacing Plan (by concept) - 164 days

Ch1	Drawing Conclusions from Data	(8 days traditional/4 block schedule)	Alg2
Section	Title	CCSS	Sect
1-1	Analyzing Data	N.Q.A.2, S.ID.A.2, S.IC.B.6	
1-2 Lab	Lesson Lab: An Intro to Summation Notation		
1-2	Standard Deviation	S.ID.A.2, S.IC.B.6	7.7
1-3	Samples & Surveys	S.ID.A.1, S.IC.B.3, S.IC.B.6	
1-4a Lab	Activity Lab: Probability Distributions	S.ID.A.2, S.MD.B.6	
1-4	Normal Distributions	S.ID.A.4, S.MD.B.7	12.7
1-4b Lab	Activity Lab: Margin of Error	S.IC.B4	
1-4c Lab	Activity Lab: Drawing Conclusions from Samples	S.IC.B5	

* Section 1-2 Lab has no CCSS's listed

* Supplement for Sections 1-1, 1-3

Ch2	Linear Equations, Inequalities, & Functions	(18 days traditional/9 block schedule)	Alg2
Section	Title	CCSS	Sect
2-1	Solving Equations	A.CED.A.1, A.CED.A.4	1.3
2-2	Solving Inequalities	A.CED.A.1	1.6
2-3	Linear Functions & Slope-Intercept From	A.CED.A.2, F.IF.B.4, F.IF.C.7a	2.4
2-4	Families of Functions	F.BF.B.3	SUPP
2-5	Absolute Value Functions & Graphs	F.IF.C.7b, F.BF.B.3	2.8
2-5 Lab	Lesson Lab: Piecewise Functions	F.IF.C.7b	
2-6	Two-Variable Inequalities	A.CED.A.3	2.6
2-7	Solving Systems of Equations	A.CED.A.2, A.CED.A.3	3.1, 3.2
2-8	Systems of Inequalities	A.CED.A.3	3.6
2-9	Systems with Three Variables	A.CED.A.3	3.5
2-10	Solving Systems Using Matrices	A.CED.A.3	4.5

* Supplement for Section 2-4

Ch3	Quadratic Functions & Equations	(16 days traditional/8 block schedule)	Alg2
Section	Title	CCSS	Sect
3-1	Quadratic Functions & Transformations	A.CED.A.2, F.IF.B.4, F.IF.B.6, F.IF.C.7b, F.BF.B.3	5.1
3-2	Standard Form of a Quadratic Function	A.CED.A.2, F.IF.B.4, F.IF.B.6, F.IF.C.9, F.BF.A.1b	5.1, 5.2
3-2 Lab	Lesson Lab: Identifying Quadratic Functions	F.IF.B.6	
3-3	Factoring Quadratic Expressions	A.SSE.A.1a, A.SSE.A.2, F.IF.C.8a	5.2
3-4	Quadratic Equations	A.SSE.A.1a, A.APR.B.3, A.CED.A.1, F.IF.B.4, F.IF.C.8a	5.3, 5.4
3-5	Completing the Square	A.SSE.A.2, A.CED.A.1, F.IF.C.8a	5.5
3-6	The Quadratic Formula	A.CED.A.1	5.6
3-7	Quadratic Systems	A.CED.A.1, A.CED.A.3, A.REI.C.7, A.REI.D.11	10.7
3-8	A New Look at Parabolas	G.GPE.A.2, G.GPE.B.4	10.6
3-9	Circles in the Coordinate Plane	G.GPE.A.1, G.GPE.B.4	10.3, 10.6

Ch4	Polynomials & Polynomial Functions	(20 days traditional/10 block schedule)	Alg2
Section	Title	CCSS	Sect
4-1	Polynomial Functions	A.SSE.A.1a, F.IF.C.7c	6.1, 6.2
4-1 Lab	Tech Lab: Even & Odd Functions	F.BF.B.3	
4-2	Adding, Subtracting & Multiplying Polynomials	A.SSE.A.2, A.APR.A.1	6.3
4-3	Polynomials, Linear Factors & Zeros	N.Q.A.2, A.SSE.A.1a, A.APR.B.2, A.APR.B.3, A.CED.A.2,	6.6
		F.IF.B.4, F.IF.C.7c, F.BF.A.1b	
4-4	Solving Polynomial Equations	N.Q.A.2, A.SSE.A.1b, A.REI.D.11	6.4
4-5	Dividing Polynomials	A.APR.B.2, A.APR.D.6	6.5
4-6	Theorems About Roots of Polynomial Equations	N.CN.C.8, A.SSE.A.2, A.APR.B.3	6.6
4-6 Lab	Lesson Lab: Using Polynomial Identities	N.CN.C.8, A.SSE.A.2, A.APR.C.4	
4-7	The Fundamental Theorem of Algebra	N.CN.C.8,N.CN.C.9, A.SSE.A.2, A.APR.B.3	6.7
4-7 Lab	Activity Lab: Graphing Polynomials Using Zeros	A.APR.B.3, F.IF.C.7c	
4-8	The Binomial Theorem	A.SSE.A.2, A.APR.C.5	12.6
4-8 Lab	Lesson Lab: Mathematical Induction	A.APR.C.5	

4-9	Polynomial Models in the Real World	N.Q.A.2, A.CED.A.2, F.IF.B.4, F.IF.B.6, F.IF.C.7c	6.8, 6.9
4-10	Transforming Polynomial Functions	F.IF.C.7c, F.IF.C.9, F.BF.B.3	6.8, 6.9

Ch5	Rational Expressions & Functions	(16 days traditional/8 block schedule)	Alg2
Section	Title	CCSS	Sect
5-1	Simplifying Rational Expressions	A.SSE.A.1b, A.SSE.A.2, A.APR.D.6	
5-2	Multiplying & Dividing Rational Expressions	A.SSE.A.2, A.APR.D.7	9.4
5-3	Adding & Subtracting Rational Expressions	N.Q.A.2, A.SSE.A.2, A.APR.D.7	9.5
5-4	Inverse Variations	A.CED.A.2, F.IF.B.4, F.IF.C.7d	9.1
5-5 Lab	Tech Lab: Graphing Rational Functions	F.IF.C.7d	
5-5	The Reciprocal Functions Family	A.SSE.A.1b, A.CED.A.2, F.IF.B.4, F.IF.B.5,	
		F.IF.C.7d, F.IF.C.8a, F.BF.A.1b, F.BF.B.3	
5-6	Rational Functions & Their Graphs	A.SSE.A.1b, A.SSE.A.2, A.CED.A.2, F.IF.B.4,	9.2, 9.3
		F.IF.B.5, F.IF.B.6, F.IF.C.7d, F.IF.C.9, F.BF.A.1b	
5-6 Lab	Tech Lab: Oblique Asymptotes	A.SSE.A.2, F.IF.C.7d	
5-7	Solving Rational Equations	N.Q.A.2, A.SSE.A.2, A.REI.A.1, A.REI.A.2, A.REI.D.11	9.6
5-7a Lab	Activity Lab: Systems with Rational Equations	A.CED.A.3, A.REI.D.11	
5-7b Lab	Tech Lab: Rational Inequalities	A.CED.A.1	

* Supplement for Sections 5-1 & 5-5

Ch6	Radical Expressions & Functions	(16 days traditional/8 block schedule)	Alg2
Section	Title	CCSS	Sect
6-1	Roots & Radical Expressions	A.SSE.A.2	7.1
6-2	Multiplying & Dividing Rational Expressions	A.SSE.A.2	7.2
6-3	Binomial Radical Expressions	A.SSE.A.2	7.1, 7.2
6-4	Rational Exponents	A.SSE.A.2	7.1, 7.2
6-5	Solving Square Root & Other Radical Equations	A.CED.A.1, A.REI.A.2	7.6
6-6	Function Operations	N.Q.A.2, F.BF.A.1c	7.3
6-7	Inverse Relations & Functions	F.BF.B.4a, F.BF.B.4c	7.4
6-8	Graphing Radical Functions	A.CED.A.2, F.IF.B.4, F.IF.B.5, F.IF.B.6, F.IF.C.7b, F.IF.C.9	7.5

Ch7	Exponential & Logarithmic Functions	(12 days traditional/6 block schedule)	Alg2
Section	Title	CCSS	Sect
7-1	Exploring Exponential Models	N.Q.A.2, A.SSE.A.1b, A.CED.A.2, F.IF.C.7e, G.GMD.A.1	8.7
7-2	Properties of Exponential Functions	A.SSE.A.1b, A.SSE.B.3c, A.CED.A.2,	8.1, 8.2
		F.IF.C.7e, F.IF.C.8b	
7-3	Logarithmic Functions as Inverses	A.SSE.A.1b, F.IF.C.7e, F.IF.C.8b, F.IF.C.9,	8.3, 8.4
		F.BF.A.1c, F.BF.B.3, F.BF.B.4a, F.BF.B.4b, F.BF.B.5	
7-4	Properties of Logarithms	A.SSE.A.2	8.5
7-5	Exponential & Logarithmic Equations	N.Q.A.2, A.CED.A.1, A.REI.D.11, F.IF.C.7e, F.BF.B.5, F.LE.A.4	8.6, 8.7
7-5 Lab	Tech Lab: Using Logarithms for Exponential Models	F.IF.C.7e, F.IF.C.8a	
7-6	Natural Logarithms	F.LE.A.4	8.6
7-6 Lab	Lesson Lab: Exponential & Logarithmic Inequalities	A.CED.A.1, A.REI.D.11	

Ch8	Trigonometric Functions	(22 days traditional/11 block schedule)	Alg2
Section	Title	CCSS	Sect
8-1	Exploring Periodic Data	F.IF.B.4	13.2
8-2	Angles and the Unit Circle		13.1, 13.2
8-3	Radian Measures	F.TF.A.1	13.1, 13.2
8-4	The Sine Function	F.IF.B.4, F.IF.B.5, F.IF.B.6, F.IF.C.7e, F.TF.A.2, F.TF.B.5	14.1
8-4 Lab	Tech Lab: Graphing Trigonometric Functions	F.IF.C.7e	
8-5	The Cosine Function	N.Q.A.2, F.IF.B.4, F.IF.C.7e, F.IF.C.9, F.TF.A.2, F.TF.B.5	14.1
8-6	The Tangent Function	F.IF.B.4, F.IF.C.7e, F.TF.A.2, F.TF.B.5	SUPP
8-7	Translating Sine & Cosine Functions	F.IF.C.7e, F.BF.B.3, F.TF.B.5	14.2
8-7 Lab	Tech Lab: Plotting & Analyzing Residuals		
8-8	Reciprocal Trigonmetric Functions	F.IF.B.4, F.IF.C.7e, F.TF.A.2, F.TF.B.5	13.3
8-9	Trigonometric Identities	F.TF.C.8	14.3
8-10	Area and the Law of Sines	G.SRT.D.9, G.SRT.D.10, G.SRT.D.11	13.5

8-10 Lab	Lesson Lab: The Ambiguous Case	G.SRT.D.11	13.5
8-11	The Law of Cosines	G.SRT.D.10, G.SRT.D.11	13.6

* Section 8-2 has no CCSS's listed * Section 8-7 Lab has no CCSS's listed

* Supplement for Section 8-6

Ch9	Sequences & Series	(8 days traditional/4 block schedule)	Alg2
Section	Title	CCSS	Sect
9-1	Mathematical Patterns	F.BF.A.1a	11.1, 11.5
9-2	Arithmetic Sequences	F.BF.A.1a	11.2
9-3	Geometric Sequences	F.BF.A.1a	11.3
9-4	Arithmetic Series	F.BF.A.1a	11.2
9-5 Lab	Activity Lab: Geometry & Infinite Series	A.SSE.B.4	11.4
9-5	Geometric Series	A.SSE.B.4	11.3, 11.4

* Section 9-6 is not listed in the Table of Contents in Pearson Book but it is listed in the Correlation of the CCSS Worksheet (typo for 9-5)

Ch10	Applying Geometric Concepts	(12 days traditional/6 block schedule)	Alg2
Section	Title	CCSS	Sect
10-1	Applying Constructions	G.CO.D.12, G.CO.D.13, G.C.A.3	SUPP
10-2	Solving Density & Design Problems	G.MG.A.2, G.MG.A.3	SUPP
10-3	Perimeters & Areas of Similar Figures	G.MG.A.1, G.MG.A.2, G.MG.A.3	SUPP
10-4	Geometric Probability	G.MG.A.1	SUPP
10-5	Space Figures & Cross-Sections	G.GMD.B.4	SUPP
10-6	Areas & Volumes of Similar Solids	G.MG.A.1, G.MG.A.2, G.MG.A.3	SUPP
10-7	Locus: A Set of Points	G.GMD.B.4	SUPP

* Supplement for Sections 10-1 thru 10-7

Ch11	Connecting Algebra and Geometry	(8 days traditional/4 block schedule)	Alg2
Section	Title	CCSS	Sect
11-1	Perimeter & Area in the Coordinate Plane	N.Q.A.1, G.GPE.B.7	SUPP
11-1 Lab	Lesson Lab: Partitioning a Segment	G.GPE.B.6	
11-2	Areas of Parallelograms & Triangles	G.GPE.B.7, G.MG.A.1	SUPP
11-3	Areas of Trapezoids, Rhombi & Kites	G.MG.A.1	SUPP
11-4 Lab	AL: Proving Slope Criteria for Parallel & Perpendicular Lines	G.GPE.B.5	
11-4	Polygons in the Coordinate Plane	G.GPE.B.4	SUPP

* Supplement for Sections 11-1 thru 11-4

Ch12	Circles	(10 days traditional/5 block schedule)	Alg2
Section	Title	CCSS	Sect
12-1	Circles & Arcs	G.C.A.1, G.C.A.2, G.C.B.5	SUPP
12-2	Areas of Circles & Sectors	G.C.B.5	SUPP
12-2 Lab	Activity Lab: Circles and Radians	G.C.B.5	
12-3	Tangent Lines	G.C.A.2, G.C.A.4	SUPP
12-4	Chords & Arcs	G.C.A.2	SUPP
12-5	Inscribed Angles	G.C.A.2, G.C.A.3, G.C.A.4	SUPP
12-6	Angle Measures & Segment Lengths	G.C.A.2	SUPP

* Supplement for Sections 12-1 thru 12-6



Mathematics III

It is in the Mathematics III course that students integrate and apply the mathematics they have learned from their earlier courses. This course includes standards from the conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Some standards are repeated in multiple higher mathematics courses; therefore instructional notes, which appear in brackets, indicate what is appropriate for study in this particular course. Standards that were limited in Mathematics I and Mathematics II no longer have those restrictions in Mathematics III.

For the Mathematics III course, instructional time should focus on four critical areas: (1) apply methods from probability and statistics to draw inferences and conclusions from data; (2) expand understanding of functions to include polynomial, rational, and radical functions; (3) expand right triangle trigonometry to include general triangles; and (4) consolidate functions and geometry to create models and solve contextual problems.

- (1) Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the roles that randomness and careful design play in the conclusions that can be drawn.
- (2) The structural similarities between the system of polynomials and the system of integers are developed. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. Rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of the Mathematics III course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. This critical area also includes exploration of the Fundamental Theorem of Algebra.
- (3) Students derive the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.
- (4) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this Mathematics III course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Note: The source of this introduction is the *Massachusetts Curriculum Framework for Mathematics* (Malden: Massachusetts Department of Elementary and Secondary Education, 2011), 147–8.

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.

Geometry

Similarity, Right Triangles, and Trigonometry

• Apply trigonometry to general triangles.

Expressing Geometric Properties with Equations

• Translate between the geometric description and the equation for a conic section.

Geometric Measurement and Dimension

• Visualize relationships between two-dimensional and three-dimensional objects.

Modeling with Geometry

• Apply geometric concepts in modeling situations.

Statistics and Probability

Interpreting Categorical and Quantitative Data

• Summarize, represent, and interpret data on a single count or measurement variable.

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Using Probability to Make Decisions

• Use probability to evaluate outcomes of decisions.



Number and Quantity

The Complex Number System

Use complex numbers in polynomial identities and equations. [Polynomials with real coefficients; apply N.CN.9 to higher degree polynomials.]

- 8. (+) Extend polynomial identities to the complex numbers.
- 9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Algebra

	Seeing Structure in Expressions A-S	SE					
Interpret the structure of expressions. [Polynomial and rational]							
	1. Interpret expressions that represent a quantity in terms of its context. \star						
	a. Interpret parts of an expression, such as terms, factors, and coefficients. \star						
	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. \star						
	. Use the structure of an expression to identify ways to rewrite it.						
	Vrite expressions in equivalent forms to solve problems.						
	. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula problems. For example, calculate mortgage payments. *	to solve					

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. [Beyond quadratic]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

- 2. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x a is p(a), so p(a) = 0 if and only if (x a) is a factor of p(x).
- 3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

- 4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
- 5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.¹

N-CN

A-APR

Note: Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.(+) Indicates additional mathematics to prepare students for advanced courses.

^{1.} The Binomial Theorem may be proven by mathematical induction or by a combinatorial argument.

Rewrite rational expressions. [Linear and guadratic denominators]

- 6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- 7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

Create equations that describe numbers or relationships. [Equations using all available types of expressions, including simple root functions]

- 1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. CA *****
- 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
- 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ★
- 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **★**

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning. [Simple radical and rational]

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Represent and solve equations and inequalities graphically. [Combine polynomial, rational, radical, absolute value, and exponential functions.]

11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. \star

Functions

Interpreting Functions

Interpret functions that arise in applications in terms of the context. [Include rational, square root and cube root; emphasize selection of appropriate models.]

 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★

Mathematics III

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- 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. ★
- 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

Analyze functions using different representations. [Include rational and radical; focus on using key features to guide selection of appropriate type of model function.]

- 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. 🖈
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. 🖈
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. *
- 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions

Build a function that models a relationship between two quantities. [Include all types of functions studied.]

- 1. Write a function that describes a relationship between two quantities. \star
 - b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. **★**

Build new functions from existing functions. [Include simple, radical, rational, and exponential functions; emphasize common effect of each transformation across function types.]

- 3. Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
- 4. Find inverse functions.
 - a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = (x + 1)/(x - 1) for $x \neq 1$.

Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

- For exponential models, express as a logarithm the solution to *ab^{ct}* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. ★ [Logarithms as solutions for exponentials]
- 4.1. Prove simple laws of logarithms. CA 🖈
- 4.2 Use the definition of logarithms to translate between logarithms in any base. CA \star

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F-LE

F-BF

4.3 Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. CA ★

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

- 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- 2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- 2.1 Graph all 6 basic trigonometric functions. CA

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. 🖈

Geometry

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles.

- 9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- 10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
- 11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

3.1 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola and graph the equation. [In Mathematics III, this standard addresses only circles and parabolas.] CA

Geometric Measurement and Dimension

Visualize relationships between two-dimensional and three-dimensional objects.

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry

Apply geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). *

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s to simplify logarithmic numeric expressions and to identify their

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M3 Mathematics III

- Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★
- 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). *

Statistics and Probability

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

- Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
- 2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? ★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

- 3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★
- Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
- 5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★
- 6. Evaluate reports based on data. \star

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. [Include more complex situations.]

- 6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). 🖈
- (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). ★



S-IC

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